Model Question
M.Sc. Sem- IV
Paper - ECMATH403(C)
Integral Equations.

Very Short answer type question.

1. Define linear integral equation.
2. Define Fredholm Integral equation.
3. Define Volterra Integral equation.
4. Define Singular Integral equation.
5. Define Symmetric Kernels.
6. Define Seperable Kernels.
7. Define Iterated kernels.

8. An integral equation in which $K(x,t) = x-t$ is known as ________.

9. Define an initial Value Problem.
10. Define Boundary Value Problem.

11. If $K_n(x,t)$ be iterated Kernels then $R(x,t;\lambda) = \Gamma(x,t;\lambda) =$ ________.

12. The solⁿ of the integral equation $x = \int_0^x e^{x-t} u(t)\,dt$ is

a) $u(x) = 1+x$ c) $u(x) = x$
b) $u(x) = 1-x$ d) $u(x) = 1$

13. $f(x) = \lambda \int_a^b K(x,t)\,\phi(t)\,dt$ is

a) FIE of first kind c) VIE of first kind
b) FIE of Second kind d) VIE of Second kind

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14. Define Integro Differential equation.

15. The Eigenvalues of a symmetric kernel are
a) Real
b) Imaginary
c) Both (a) and (b)
d) none of the above

16. Two functions $f_1(x)$ and $f_2(x)$, continuous on the interval $[a, b]$ are said to be orthogonal on $[a, b]$ if ______

17. For a FIE of second kind, if $D(\lambda) \neq 0$ then the FIE of second kind has.
1). No Solution
2) Zero Solution
3) Infinite Solution
4) unique non-zero sol^n

18. The number of eigen functions is known as ______ of the eigen values.

19. Write Leibnitz's Rule.

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Short answer type questions.

1. Convert the following differential equation into an integral equation
$$y'' + \lambda xy = f(x) \; ; \; y(0)=1, \; y'(0)=0.$$

2. Form an Integral equation corresponding to the differential equation.
$$y''' + xy'' + (x^2 - x)y = xe^x + 1$$
with initial conditions.
$$y(0) = 1 = y'(0), \quad y''(0) = 0.$$

3. Prove that all iterated Kernels of a symmetric Kernel are also symmetric

4. Prove that the eigen function of a symmetric Kernel corresponding to different eigen values are orthogonal

5. If $K(x,t)$ is real and symmetric, continuous and identically not equal to zero, then Prove that all the characteristics constants are real.

6. Solve the following homogeneous Fredholm Integral equation using Schmidt solution.
$$f(x) = \lambda \int_0^1 e^x e^t f(t)\, dt$$

7. Find the iterated Kernel of the function
$$K(x,t) = e^x \cos t \; ; \; a=0, \; b=\pi$$

8 Find the resolvent Kernel of the funᶜ
$$K(x,t) = (1+x)(1-t), \quad a=-1, \; b=0$$

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9 Show that the function $u(x) = 1 - x$ is a solution of the integral equation

$$\int_0^x e^{x-t}\, u(t)\, dt = x$$

10. Find the eigen values and eigen functions of the homogeneous integral equation

$$u(x) = \lambda \int_0^1 e^x e^t\, u(t)\, dt$$

Long answer type questions

1. Show that $y(x) = xe^{x}$ is a solution of the VIE
$$y(x) = \sin x + 2\int_0^x \cos(x-t)\, y(t)\, dt.$$

2. Show that $y(x) = \cos 2x$ is a solution of the integral equation
$$y(x) = \cos x + 3\int_0^{\pi} K(x,t)\, y(t)\, dt$$
where,
$$K(x,t) = \begin{cases} \sin x \cdot \cos t & ; 0 \le x \le t \\ \cos x \cdot \sin t & t \le x \le \pi \end{cases}$$

3. Solve the homogeneous Fredholm integral equation of the second kind
$$y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)\, dt$$

4. Solve the following integral equation
$$y(x) = x + \lambda \int_0^1 (1+x+t)\, y(t)\, dt$$

5. Solve
$$y(x) = x + \lambda \int_0^1 (xt^2 + tx^2)\, y(t)\, dt$$

6. Solve
$$y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, y(t)\, dt$$

$\boxed{\text{or}}$

obtain FIE of second kind corresponding to the Boundary value problem is
$$\frac{d^2u}{dx^2} + \lambda u = x \quad \text{with Boundary Conditions}$$
$u(0) = 0$, $u'(1) = 0$. Also, recover the BVP from IE you obtain.

7. Consider the Volterra Integral equation of the second kind

$$u(x) = f(x) + \lambda \int_a^x K(x,t)\, u(t)\, dt$$

Also suppose that

i) Kernel $K(x,t) \neq 0$ is real and continuous in a rectangle R $(a \le x \le b,\ a \le t \le b)$

ii) Also,
$K(x,t) \le M$ on R, i.e. $K(x,t)$ is bounded by M on R

ii) $f(x) \neq 0$ is real and continuous in the interval I $(a \le x \le b)$
Also let $|f(x)| \le N$ in I.

iii) λ is a constant
Then prove that given VIE has a unique continuous solution in I, which is given by the absolutely and uniformly series.

$$u(x) = f(x) + \lambda \int_a^x K(x,t) f(t)\, dt + \lambda^2 \int_a^x K(x,t) \int_a^t K(t,t_1) f(t_1)\, dt_1\, dt + \cdots$$

(Sol^n of VIE of 2nd kind by successive substitution)

8. By means of Resolvent kernel, find the solution of the integral equation

$$u(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2}\, u(t)\, dt$$

9. Find the sol^n of Abel's integral equation.

10. Solve the integral equation

$$f(x) = \int_a^x \frac{y(t)\, dt}{(\cos t - \cos x)^{1/2}} \qquad 0 \le a < x < b \le \pi$$